
# Chapter 6

## The Meaning and Measurement of Risk and Return

# Learning Objectives

- Define and measure the expected rate of return of an individual investment.
- Define and measure the riskiness of an individual investment.
- Compare the historical relationship between risk and rates of return in the capital markets.

# Learning Objectives

- Explain how diversifying investments affects the riskiness and expected rate of return of a portfolio or combination of assets.
- Explain the relationship between an investor's required rate of return on an investment and the riskiness of the investment.

# EXPECTED RETURN DEFINED AND MEASURED

# Holding-Period Return

Historical or holding-period or realized rate of return

Holding-period return = payoff during the "holding" period. Holding period could be any unit of time such as one day, few weeks or few years.

$$\text{Holding-period dollar gain, DG} = \text{price}_{\text{end of period}} + \text{cash distribution (dividend)} - \text{price}_{\text{beginning of period}} \quad (6\text{-}1)$$

# Holding-Period Return

- You bought 1 share of Google for $524.05 on April 17 and sold it one week later for $565.06. Assuming no dividends were paid, your dollar gain was:

  565.06 – 524.05 = $41.01

# Holding-Period Rate of Return

Holding-period rate of return

$$\text{Rate of return, } r = \frac{\text{dollar gain}}{P_{\text{beginning of period}}} = \frac{P_{\text{end of period}} + \text{Dividend} - P_{\text{beginning of period}}}{P_{\text{beginning of period}}}$$

Google rate of return:

41.01/524.05 = .0783 or 7.83%

# Expected Return

Expected Cash Flows and Expected Rate of Return

- The expected benefits or returns an investment generates come in the form of cash flows.
- Cash flows are used to measure returns (not accounting profits).

# Expected Return

- The expected cash flow is the weighted average of the *possible* cash flows outcomes such that the weights are the probabilities of the occurrence of the various states of the economy.
- Expected Cash flow ($X$) = $\Sigma Pb_i * CF_i$
  - Where $Pb_i$ = probabilities of outcome $i$
  - $CF_i$ = cash flows in outcome $i$

**TABLE 6-1** Measuring the Expected Return of an Investment

| State of the Economy | Probability of the States[a] | Cash Flows from the Investment | Percentage Returns (Cash Flow ÷ Investment Cost) |
|---|---|---|---|
| Economic recession | 20% | $1,000 | 10% ($1,000 ÷ $10,000) |
| Moderate economic growth | 30% | 1,200 | 12% ($1,200 ÷ $10,000) |
| Strong economic growth | 50% | 1,400 | 14% ($1,400 ÷ $10,000) |

[a]The probabilities assigned to the three possible economic conditions have to be determined subjectively, which requires managers to have a thorough understanding of both the investment cash flows and the general economy.

# Expected Cash Flow Equation
# Equation 6-3

$$\text{Expected cash flow, } \overline{CF} = \begin{pmatrix} \text{cash flow} & & \text{probability} \\ \text{in state 1} & \times & \text{of state 1} \\ (CF_1) & & (Pb_1) \end{pmatrix} + \begin{pmatrix} \text{cash flow} & & \text{probability} \\ \text{in state 2} & \times & \text{of state 2} \\ (CF_2) & & (Pb_2) \end{pmatrix} + \ldots + \begin{pmatrix} \text{cash flow} & & \text{probability} \\ \text{in state } n & \times & \text{of state } n \\ (CF_n) & & (Pb_n) \end{pmatrix} \quad (6\text{-}3)$$

# Expected Cash Flow

- Expected Cash flow = $\Sigma Pb_i * CF_i$

  = 0.2*1000 + 0.3*1200 + 0.5*1400

  = **$1,260** on $1,000 investment

# Expected Rate of Return

- We can also determine the % expected return on $1,000 investment. Expected Return is the weighted average of all the possible returns, weighted by the probability that each return will occur.
- Expected Return (%) = $\Sigma Pb_i * r_i$

  where

  $Pb_i$ = probabilities of outcome $i$

  $r_i$ = expected % return in outcome $i$

# Expected Rate of Return Equation 6-4

$$\text{Expected rate of return, } \bar{r} = \begin{pmatrix} \text{rate of return} & & \text{probability} \\ \text{for state 1} & \times & \text{of state 1} \\ (r_1) & & (Pb_1) \end{pmatrix} + \begin{pmatrix} \text{rate of return} & & \text{probability} \\ \text{for state 2} & \times & \text{of state 2} \\ (r_2) & & (Pb_2) \end{pmatrix}$$

$$+ \ldots + \begin{pmatrix} \text{rate of return} & & \text{probability} \\ \text{for state } n & \times & \text{of state } n \\ (r_n) & & (Pb_n) \end{pmatrix} \qquad (6\text{-}4)$$

# Expected Rate of Return

- Expected Return (%) = $\Sigma Pb_i * r_i$

  where

  $P_i$ = probabilities of outcome *i*

  $k_i$ = expected % return in outcome *I*

  = 0.2(10%) + 0.3(12%) + 0.5(14%)

  = **12.6%**

# RISK DEFINED AND MEASURED

# Risk

Three important questions:

1. What is risk?
2. How do we measure risk?
3. Will diversification reduce the risk of portfolio?

# Risk Defined

- Risk refers to potential variability in future cash flows.
- The wider the range of possible future events that can occur, the greater the risk.
- Thus, the returns on common stock are more risky than returns from investing in a savings account in a bank.

# Risk Measured

Consider two investment options:

1. Invest in Treasury bond that offers a 2% annual return.
2. Invest in stock of a local publishing company with an expected return of 14% based on the payoffs (given on next slide).

# Probability of Payoffs

## Stock

| Chance of Occurrence | Rate of Return on Investment |
| --- | --- |
| 1 chance in 10 (10% chance) | –10% |
| 2 chances in 10 (20% chance) | 5% |
| 4 chances in 10 (40% chance) | 15% |
| 2 chances in 10 (20% chance) | 25% |
| 1 chance in 10 (10% chance) | 30% |

## Treasury Bond

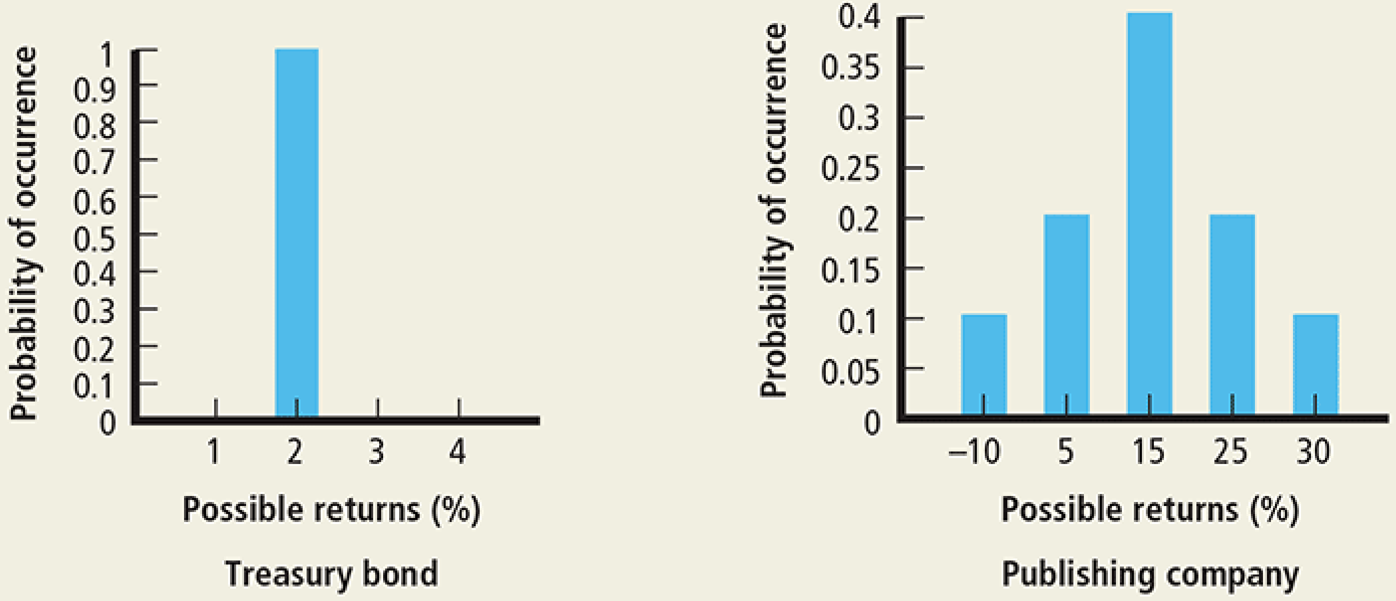
100% chance 2%

# Expected Rate of Return

- Treasury bond = 1*2% = 2%

- Stock
  = 0.1*-10 + 0.2*5% + 0.4*15% + 0.2*25% + 0.1*30%
  = 14%

**FIGURE 6-1** The Probability Distribution of the Returns on Two Investments



# Figure 6-1
# Treasury Bond versus Stock

- We observe from Figure 6-1 that the stock of the publishing company is more risky but it also offers the potential of a higher payoff.

# Standard Deviation (S.D.)

- Standard deviation (S.D.) is one way to measure risk. It measures the volatility or riskiness of portfolio returns.
- S.D. = square root of the weighted average squared deviation of each possible return from the expected return.

$$\sigma = \begin{bmatrix} (-10\% - 14\%)^2(0.10) + (5\% - 14\%)^2(0.20) \\ + (15\% - 14\%)^2(0.40) + (25\% - 14\%)^2(0.20) \\ + (30\% - 14\%)^2(0.10) \end{bmatrix}^{\frac{1}{2}}$$

$$= \sqrt{124\%} = 11.14\%$$

## TABLE 6-2 Measuring the Variance and Standard Deviation of the Publishing Company Investment

| State of the World | Rate of Return | Chance or Probability | Step 1 | Step 2 | Step 3 |
|---|---|---|---|---|---|
| A | B | C | $D = B \times C$ | $E = (B - \bar{r})^2$ | $F = E \times C$ |
| 1 | −10% | 0.10 | −1% | 576% | 57.6% |
| 3 | 5% | 0.20 | 1% | 81% | 16.2% |
| 4 | 15% | 0.40 | 6% | 1% | 0.40% |
| 5 | 25% | 0.20 | 5% | 121% | 24.2% |
| 6 | 30% | 0.10 | 3% | 256% | 25.6% |

**Step 1:** Expected Return ($\bar{r}$) = ⟶ 14%

**Step 4:** Variance = ⟶ 124%

**Step 5:** Standard Deviation = ⟶ 11.14%

# Comments on S.D.

- There is a 66.67% probability that the actual returns will fall between 2.86% and 25.14% (= 14% ± 11.14%). So actual returns are far from certain!
- Risk is relative; to judge whether 11.14% is high or low risk, we need to compare the S.D. of this stock to the S.D. of other investment alternatives.
- To get the full picture, we need to consider not only the S.D. but also the expected return.
- The choice of a particular investment depends on the investor's attitude toward risk.

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# RATES OF RETURN: THE INVESTOR’S EXPERIENCE

## FIGURE 6-2 Historical Rates of Return

| Securities | Nominal Average Annual Returns | Standard Deviation of Returns | Real Average Annual Returns[a] | Risk Premium[b] |
|---|---|---|---|---|
| Small-company stocks | 16.7% | 32.1% | 13.8% | 13.2% |
| Large-company stocks | 12.1% | 20.1% | 9.2% | 8.6% |
| Intermediate-term government bonds | 6.3% | 5.6% | 3.4% | 2.8% |
| Corporate bonds | 6.1% | 8.4% | 3.2% | 2.6% |
| U.S. Treasury bills | 3.5% | 3.1% | 0.6% | 0.0% |
| Inflation | 2.9% | 4.1% | | |

[a]The real return equals the nominal returns less the inflation rate of 2.9 percent.

[b]The risk premium equals the nominal security return less the average risk-free rate (Treasury bills) of 3.5 percent.

Source: Data from Summary Statistics of Annual Total Returns: 1926 to 2014 Yearbook, Ibbotson Associates Inc.

# Rates of Return: The Investor's Experience (1926–2014)

Figure 6-2 shows:

A. The direct relationship between risk and return

B. Only common stocks provide a reasonable hedge against inflation.

The study also observed that between 1926 and 2014, large stocks had negative returns in 22 of 86 years, while Treasury bills generated negative returns in only 1 year.

# RISK AND DIVERSIFICATION

# Portfolio

- Portfolio refers to combining several assets.

- Examples of portfolio:
  - Investing in multiple financial assets (stocks – $6000, bonds – $3000, T-bills – $1000)
  - Investing in multiple items from a single market (example:  investing in 30 different stocks)

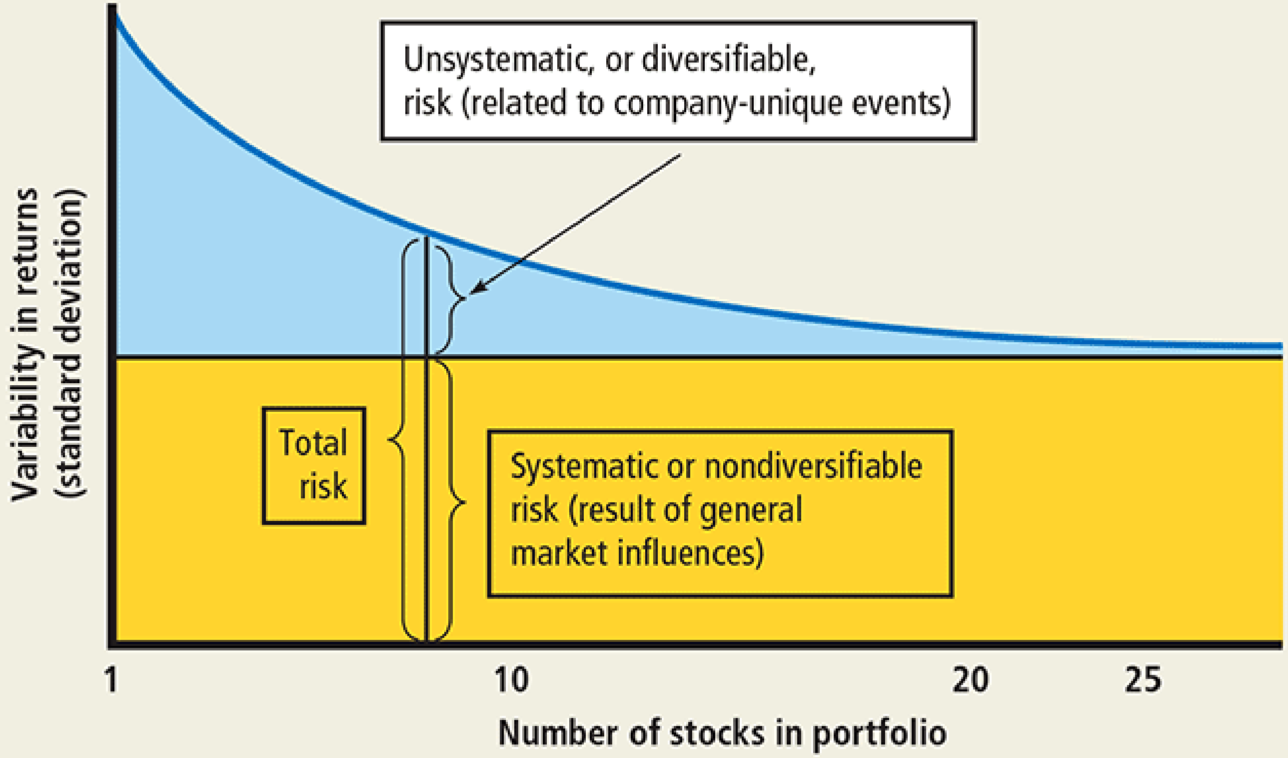
# Risk and Diversification

- Total risk of portfolio is due to two types of risk:
  - Systematic (or market risk) is risk that affects all firms (ex.: tax rate changes, war)
  - Unsystematic (or company-unique risk) is risk that affects only a specific firm (ex.: labor strikes, CEO change)

- Only unsystematic risk can be reduced or eliminated through effective diversification. (Figure 6-3)

# Total Risk & Unsystematic Risk Decline as Securities Are Added

FIGURE 6-3 Variability of Returns Compared with Size of Portfolio



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# Correlation and Risk Reduction

- The main motive for holding multiple assets or creating a portfolio of stocks (called diversification) is to reduce the overall risk exposure. The degree of reduction depends on the correlation among the assets.
  - If two stocks are perfectly positively correlated, diversification has no effect on risk.
  - If two stocks are perfectly negatively correlated, the portfolio is perfectly diversified.

# Correlation and Risk Reduction

- Thus, while building a portfolio, we should pick securities/assets that have negative or low-positive correlation to realize diversification benefits.

# Market Risk or Systematic Risk

Measuring Market Risk:

eBay vs. S&P 500

Table 6-3 and Figure 6-4 display the monthly returns for eBay and S&P 500 for the 12 months ending May 2015.

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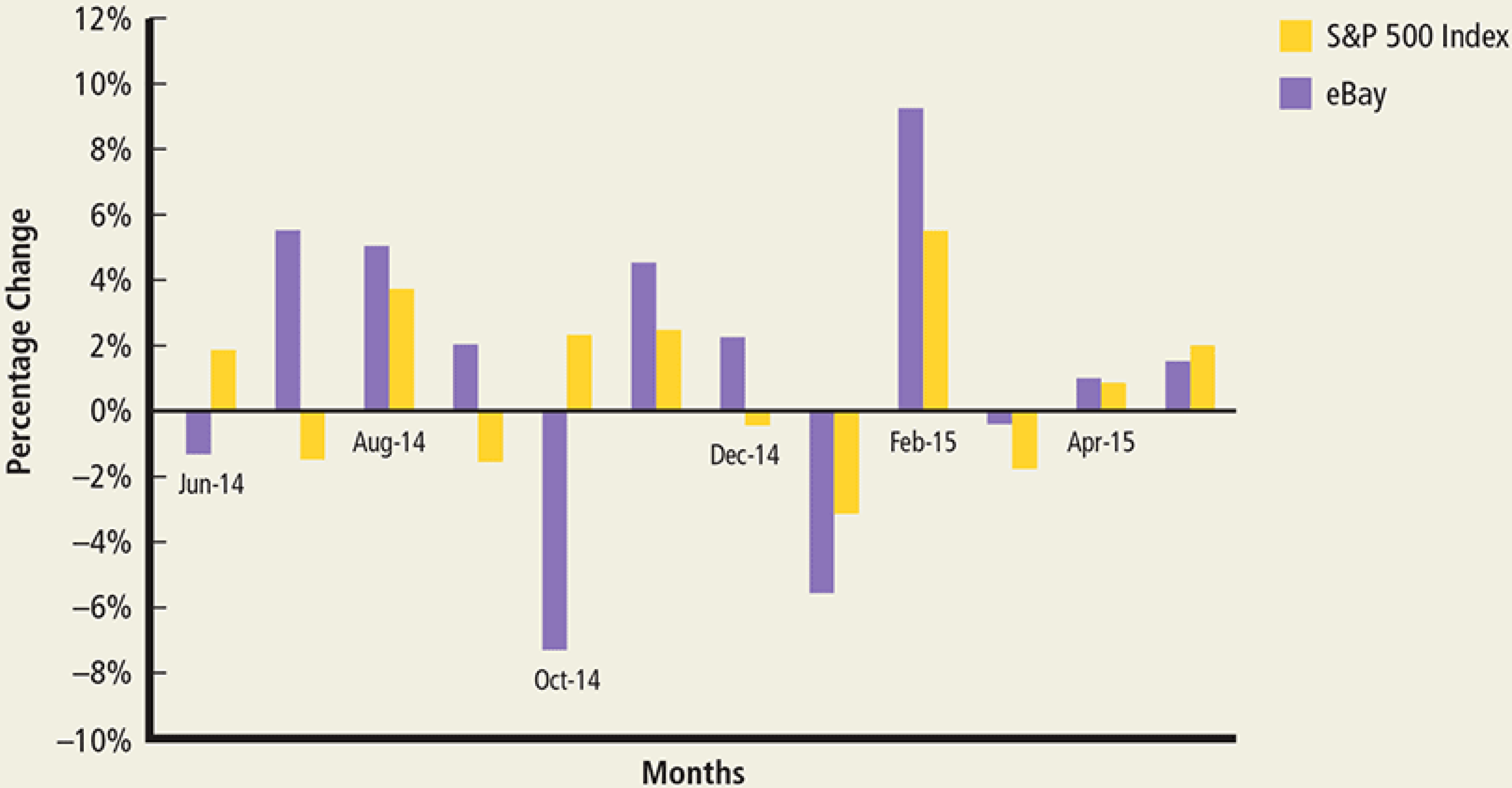
**TABLE 6-3** Monthly Holding-Period Returns, eBay versus the S&P 500 Index, June 2014 through May 2015

| Month and Year | eBay | | S&P 500 Index | |
|---|---|---|---|---|
| | Price | Returns (%) | Price | Returns (%) |
| **2014** | | | | |
| May | $50.73 | | $1,924 | |
| June | 50.06 | −1.32% | 1,960 | 1.87% |
| July | 52.83 | 5.53% | 1,931 | −1.48% |
| August | 55.50 | 5.05% | 2,003 | 3.73% |
| September | 56.63 | 2.04% | 1,972 | −1.55% |
| October | 52.50 | −7.29% | 2,018 | 2.33% |
| November | 54.88 | 4.53% | 2,068 | 2.48% |
| December | 56.12 | 2.26% | 2,059 | −0.44% |
| **2015** | | | | |
| January | 53.00 | −5.56% | 1,995 | −3.11% |
| February | 57.91 | 9.26% | 2,105 | 5.51% |
| March | 57.68 | −0.40% | 2,068 | −1.76% |
| April | 58.26 | 1.01% | 2,086 | 0.87% |
| May | 59.15 | 1.53% | 2,128 | 2.01% |
| Average Monthly Return | | 1.39% | | 0.87% |
| Standard Deviation | | 4.65% | | 2.57% |

# FIGURE 6-4 Monthly Holding-Period Returns, eBay versus the S&P 500 Index, June 2014 through May 2015




Source: Data from Yahoo Finance

# Measuring Market Risk Equation 6-6

$$\text{Monthly holding return} = \frac{\text{price}_{\text{end of month}} - \text{price}_{\text{beginning of month}}}{\text{price}_{\text{beginning of month}}}$$

$$= \frac{\text{price}_{\text{end of month}}}{\text{price}_{\text{beginning of month}}} - 1 \qquad (6\text{-}6)$$

# Equations 6-7 and 6-8

$$r_1 = \frac{P_t + D_t}{P_{t-1}} - 1 \qquad (6\text{-}7)$$

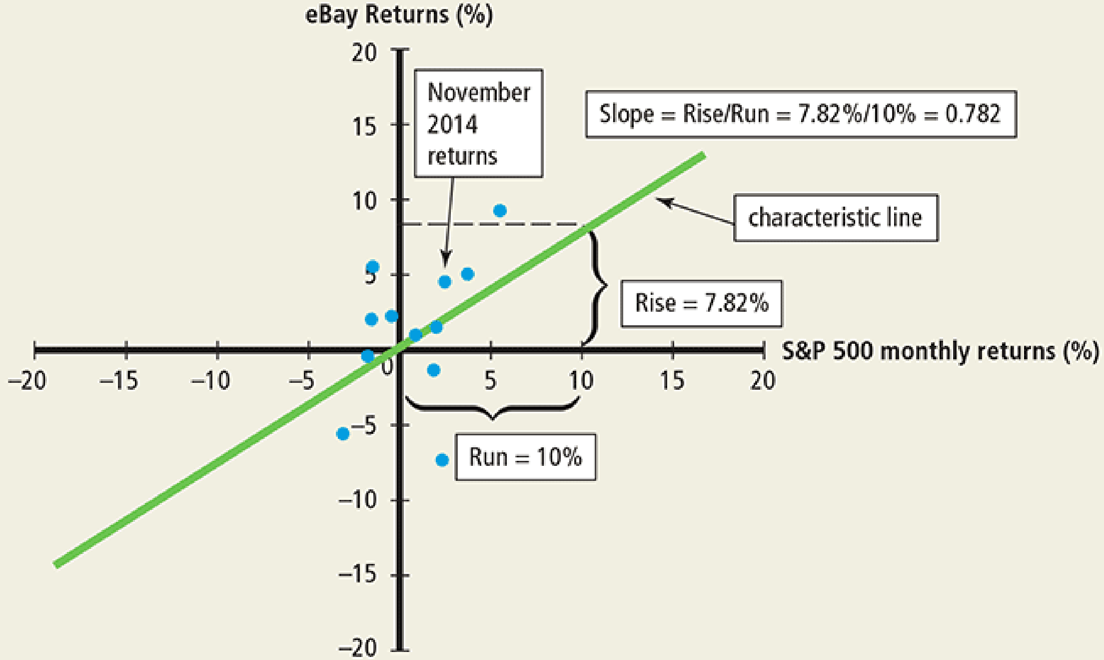
$$\text{Average holding-period return} = \frac{\text{return in month 1} + \text{return in month 2} + \cdots + \text{return in last month}}{\text{number of monthly returns}} \qquad (6\text{-}8)$$

# From Figure 6-4 and Table 6-3

- Average monthly return: eBay= 1.39%
- S&P 500 = 0.87%
- Risk was higher for eBay with standard deviation of 4.65% versus 2.57% for S&P 500.
- There is a moderate positive relationship in the movement of returns between eBay and S&P 500 (in 7 of the 12 months) (see Figure 6-5).

## FIGURE 6-5 Monthly Holding-Period Returns, eBay versus the S&P 500 Index, June 2014 through May 2015




Calculating beta:

*Visual*—the slope of a straight line can be estimated visually by drawing a straight line that best "fits" the scatter of eBay's stock returns and those of the market index. The beta coefficient then is the "rise over the run." For example, when the S&P 500 Index is 10 percent, shown on the horizontal axis (the run), eBay shown on the vertical axis (the rise) is 7.82%. Thus, beta is the rise divided by the run, or 7.82% ÷ 10%.

*Financial calculator*—financial calculators have built in functions for computing the beta coefficient. However, since the procedure varies from one calculator to another, we do not present it here.

*Excel*—Excel's Slope function can be used to calculate the slope, =slope(return values for eBay, return values for S&P).

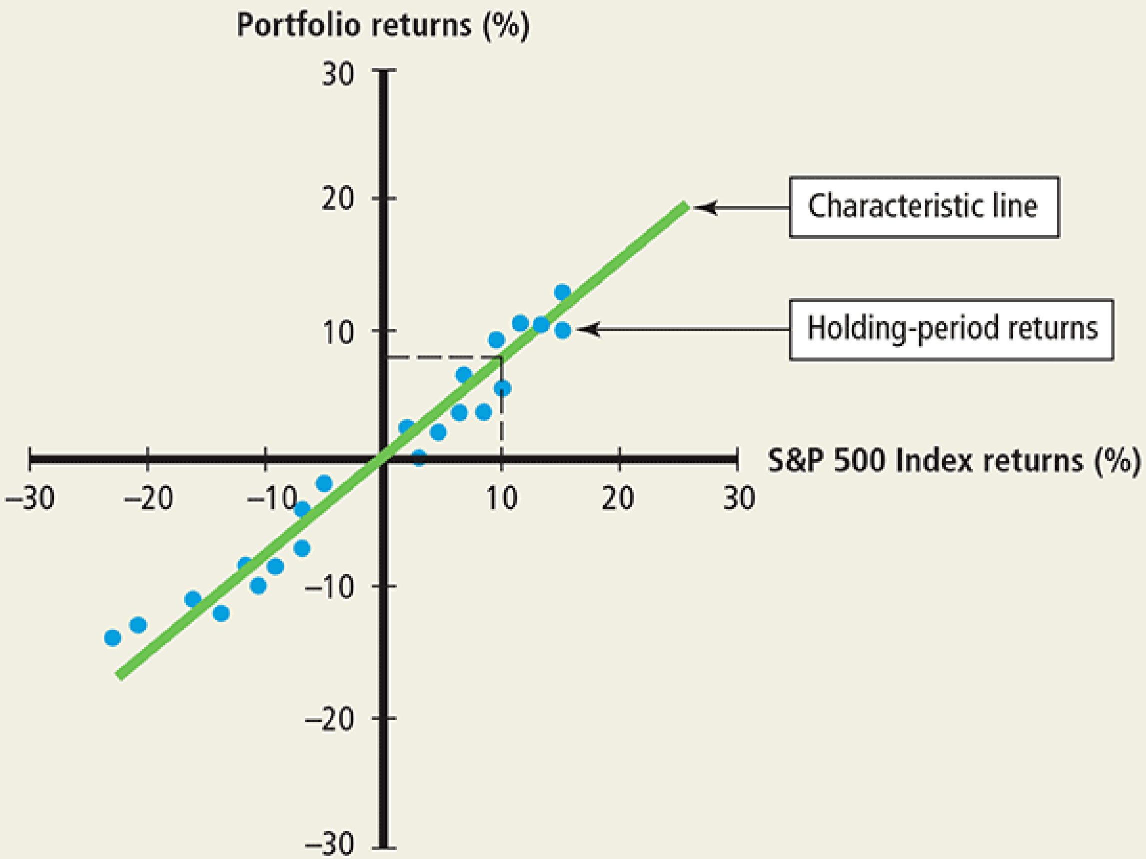
Source: Data from Yahoo! Finance.

# Characteristic Line and Beta

- The relationship between eBay and S&P 500 is captured in Figure 6-5.
- **Characteristic line** is the "line of best fit" for all the stock returns relative to returns of S&P 500.
- The slope of the characteristic line (= 0.782) measures the average relationship between a stock's returns and those of the S&P 500 Index Returns. This slope (called **beta**) is a measure of the firm's market risk; i.e., eBay's returns are 0.782 times as volatile on average as those of the overall market.

## FIGURE 6-6 Holding-Period Returns for a Hypothetical Portfolio and the S&P 500 Index



# Interpreting Beta

- Beta is the risk that remains for a company even after we have diversified our portfolio.
  - A stock with a Beta of 0 has no systematic risk
  - A stock with a Beta of 1 has systematic risk equal to the "typical" stock in the marketplace
  - A stock with a Beta exceeding 1 has systematic risk greater than the "typical" stock
- Most stocks have betas between 0.60 and 1.60. Note, the value of beta is highly dependent on the methodology and data used.

# Portfolio Beta

- Portfolio beta indicates the percentage change on average of the portfolio for every 1 percent change in the general market.

$$\beta_{\text{portfolio}} = \Sigma\, w_j * \beta_j$$

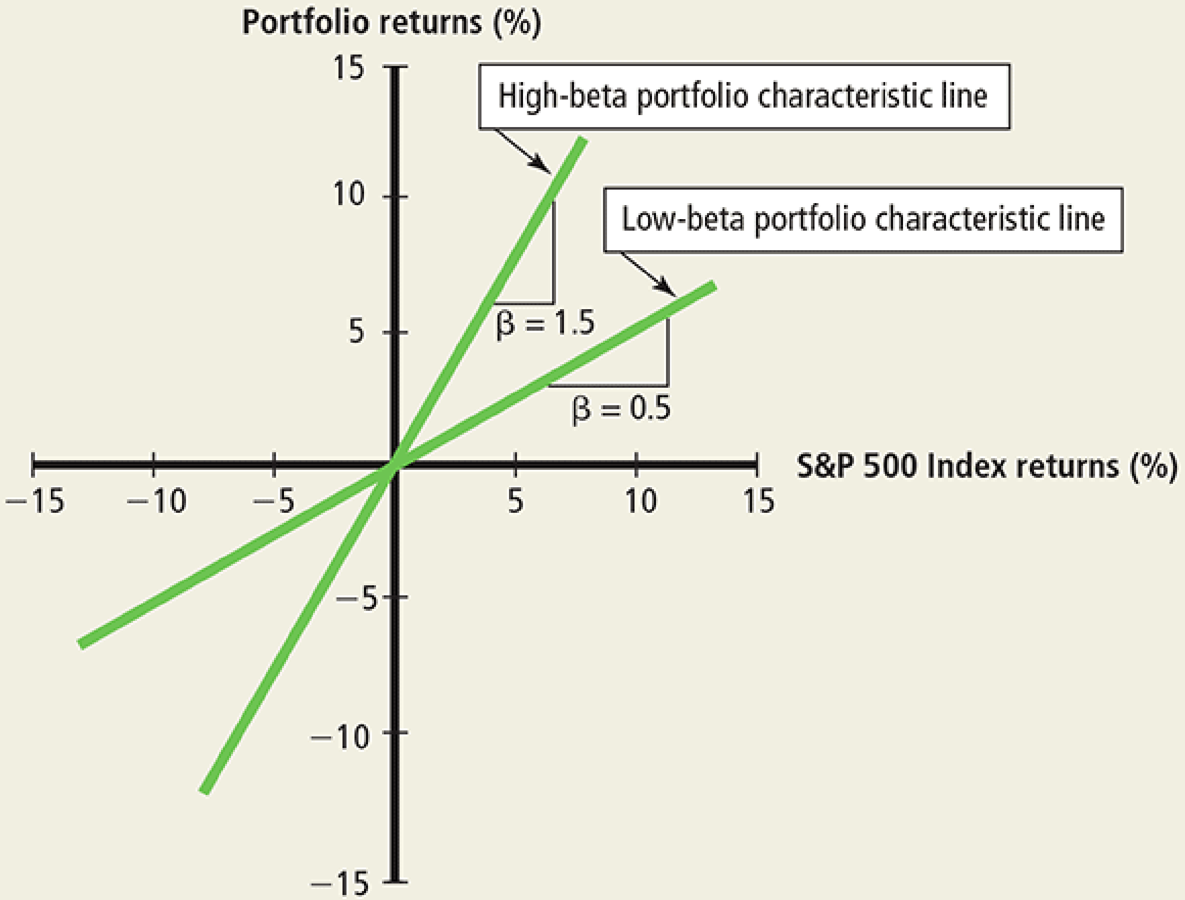
Where $w_j$ = % invested in stock $j$

$\beta_i$ = Beta of stock $j$

# Equation 6-10

$$\text{Portfolio beta} = \left(\begin{array}{c}\text{percentage of}\\ \text{portfolio invested}\\ \text{in asset 1}\end{array} \times \begin{array}{c}\text{beta for}\\ \text{asset 1}\\ (b_1)\end{array}\right) + \left(\begin{array}{c}\text{percentage of}\\ \text{portfolio invested}\\ \text{in asset 2}\end{array} \times \begin{array}{c}\text{beta for}\\ \text{asset 2}\\ (b_2)\end{array}\right) + \ldots + \left(\begin{array}{c}\text{percentage of}\\ \text{portfolio invested}\\ \text{in asset } n\end{array} \times \begin{array}{c}\text{beta for}\\ \text{asset } n\\ (b_n)\end{array}\right) \quad (6\text{-}10)$$

# FIGURE 6-7 Holding-Period Returns: High- and Low-Beta Portfolios and the S&P 500 Index



# Risk and Diversification Demonstrated

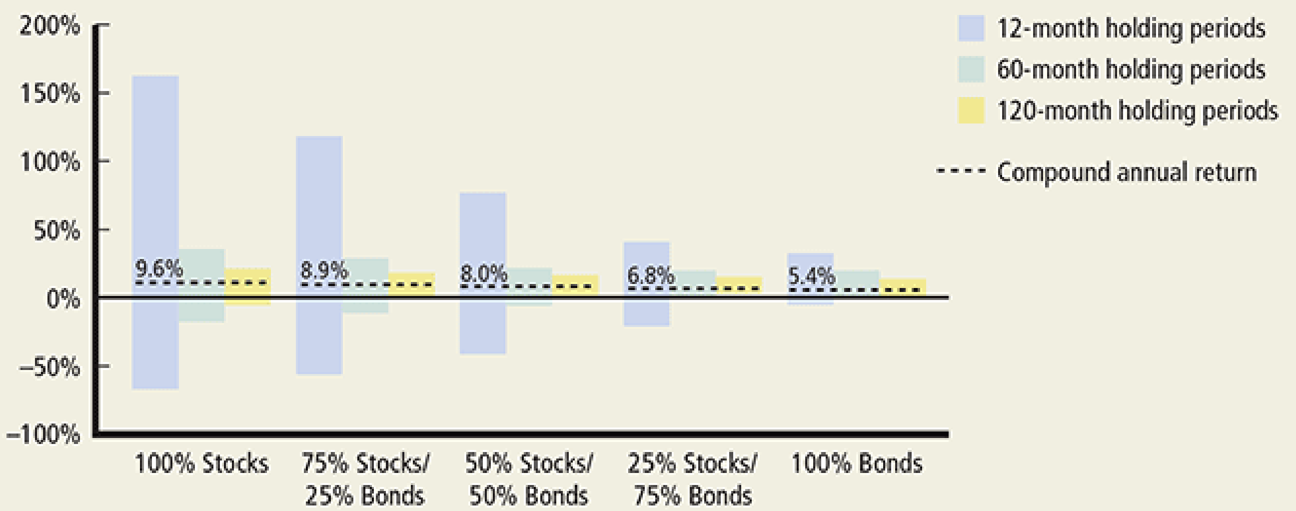
- The market rewards diversification.
- Through effective diversification, we can lower risk without sacrificing expected returns and we can increase expected returns without having to assume more risk.

# Asset Allocation

- Asset allocation refers to diversifying among different kinds of asset types (such as treasury bills, corporate bonds, common stocks).
- Asset allocation decision has to be made today – the payoff in the future will depend on the mix chosen before, which cannot be changed. Hence asset allocation decision is considered the "most important decision" while managing an investment portfolio.

**FIGURE 6-8** The Effect of Diversifying and Investing for Longer Periods of Time on Risk and Returns




Source: Data from Summary Statistics of Annual Total Returns: 1926 to 2011 Yearbook, Ibbotson Associates, Inc.

# Asset Allocation Matters!

We observe the following from Figure 6-8.

- Direct relationship between risk and return: As we move from an all-stock portfolio to a mix of stocks and bonds to an all-bond portfolio, both risk and return decline.
- Holding period matters: As we increase the holding period, risk declines.

# Asset Allocation Summary

- There has *never* been a time when investors lost money if they held an all-stock portfolio—the most risky portfolio—for 10 years.

*The market rewards the patient investor.*

# THE INVESTOR'S REQUIRED RATE OF RETURN

# The Investor's Required Rate of Return

- Investor's required rate of return is the minimum rate of return necessary to attract an investor to purchase or hold a security.
- This definition considers the opportunity cost of funds, i.e., the foregone return on the next best investment.

# The Investor's Required Rate of Return Equation 6-11

$$\text{Investor's required rate of return} = \text{risk-free rate of return} + \text{risk premum} \qquad (6\text{-}11)$$

# Risk-Free Rate

- This is the required rate of return or discount rate for risk-free investments.
- Risk-free rate is typically measured by the U.S. Treasury bill rate.

# Risk Premium

- The risk premium is the additional return we must expect to receive for assuming risk.

- As the level of risk increases, we will demand additional expected returns.

# Capital Asset Pricing Model (CAPM)

- CAPM equation equates the expected rate of return on a stock to the risk-free rate plus a risk premium for the systematic risk.

- CAPM provides for an intuitive approach for thinking about the return that an investor should require on an investment, given the asset's systematic or market risk.

# Measuring the Required Rate of Return Equation 6-12

$$\text{Risk premium} = \text{investor's required rate of return, } r - \text{risk-free rate of return, } r_f \qquad (6\text{-}12)$$

# Capital Asset Pricing Model

- If the required rate of return for the market portfolio $r_m$ is 10%, and the $r_f$ is 3%, the risk premium for the market would be 7%.
- This 7% risk premium would apply to any security having systematic (nondiversifiable) risk equivalent to the general market, or beta of 1.
- In the same market, a security with beta of 2 would provide a risk premium of 14%.

# CAPM
# Equation 6-13

CAPM suggests that beta is a factor in determining the required returns.

$$\text{Required return on security, } r = \text{risk free rate of return, } r_f + \left[\text{beta for security, b} \times \left(\text{required return on the market portfolio, } r_m - \text{risk-free rate of return, } r_f\right)\right] \quad (6\text{-}13)$$

# CAPM Example

Market risk = 10%

Risk-free rate = 3%

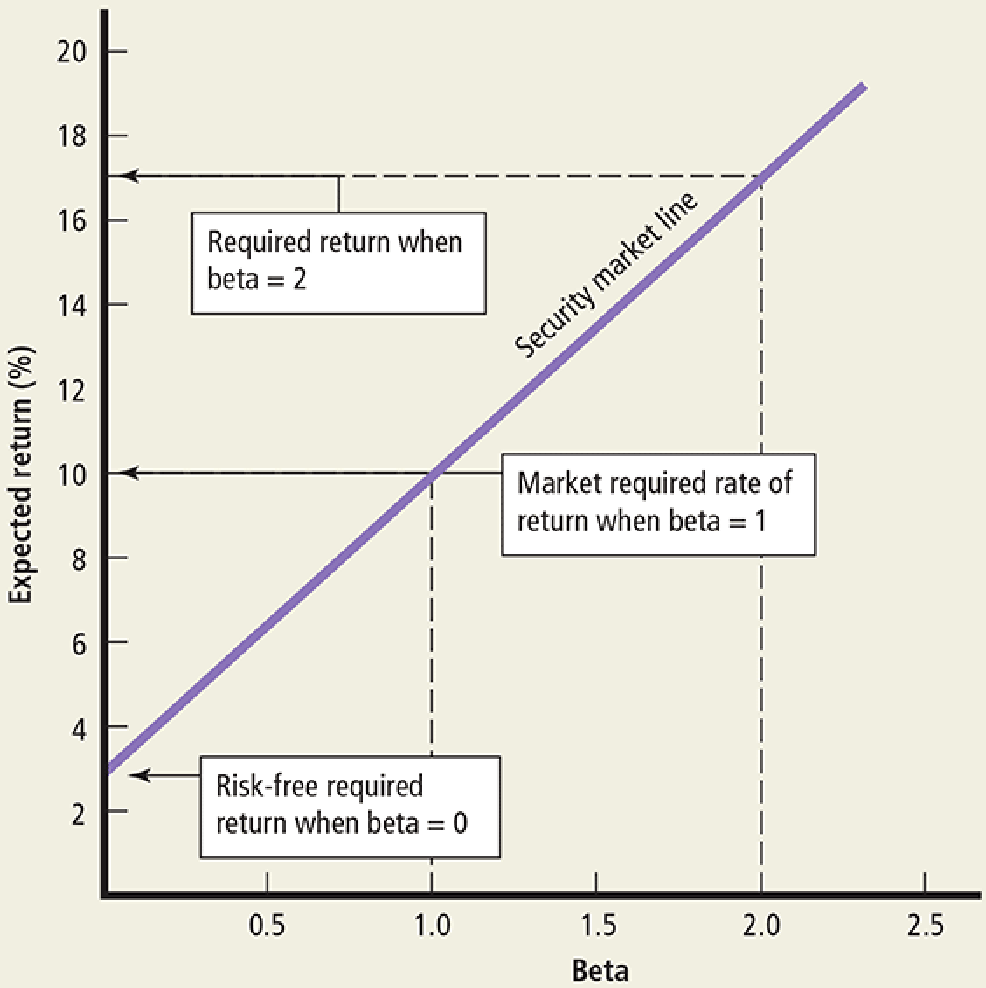
Required return = 3% + beta * (10% - 3%)

| | |
|---|---|
| If beta = 0 | Required return = 3% |
| If beta = 1 | Required return = 10% |
| If beta = 2 | Required return = 17% |

# The Security Market Line (SML)

- SML is a graphic representation of the CAPM, where the line shows the appropriate required rate of return for a given stock's systematic risk.

## FIGURE 6-9 Security Market Line



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# Key Terms

- Asset allocation
- Beta
- Capital asset pricing model (CAPM)
- Characteristic line
- Expected rate of return
- Historical or realized rate of return
- Holding-period return
- Portfolio beta
- Required rate of return
- Risk
- Risk-free rate of return
- Risk premium
- Security market line
- Standard deviation
- Systematic risk
- Unsystematic risk