CLT, LLN, CMT, SLUTZKY

- CLT: $\sqrt{n}\,(\bar{Y}_n - \underbrace{E(Y_i)}_{=\mu_Y}) \xrightarrow{d} \mathcal{N}(0, \sigma_Y^2)$

convergence in distribution

- LLN: $\bar{X}_n \xrightarrow{p} \underbrace{E(X_i)}_{\mu_X}$

convergence in probability

- CMT: $X_n \xrightarrow{p} c$ then $g(X_n) \xrightarrow{p} g(c)$
  for continuous $g(\cdot)$.

Ex: $\underbrace{\bar{Y}_n}_{X_n} \xrightarrow{p} \underbrace{\mu_Y}_{c}$ by LLN. Q: $(\underbrace{\bar{Y}_n}_{g(x)=x^2})^2 \xrightarrow{p} ?$

A: $\underbrace{(\bar{Y}_n)^2}_{g(X_n)} \xrightarrow{p} \underbrace{\mu_Y^2}_{g(c)=c^2}$

CMT also states that if $(X_n, Y_n) \xrightarrow{p} (a, b)$
then $X_n + Y_n \xrightarrow{p} a + b$, $X_n / Y_n \xrightarrow{p} a/b$
etc...

Slutzky Thm: If $X_n \xrightarrow{p} \underbrace{c}_{\text{constant}}$ and $Y_n \xrightarrow{d} \underbrace{Y}_{\text{r.v.}}$

Then,
$$X_n + Y_n \xrightarrow{d} c + Y$$
$$X_n Y_n \xrightarrow{d} cY$$
$$Y_n / X_n \xrightarrow{d} Y/c \quad \text{if } c \neq 0$$

Example: $\bar{X}_n = \frac{1}{n} \sum_{i=1}^{n} X_i$, $\bar{Y}_n = \frac{1}{n} \sum_{i=1}^{n} Y_i$

$$\bar{X}_n + \bar{Y}_n \xrightarrow{p} E(X_i) + E(Y_i) \quad \text{by CMT}$$

$$\sqrt{n}(\bar{Y}_n - \mu_Y) + \bar{X}_n \xrightarrow{d} N(0, \sigma_Y^2) + \mu_X$$

by Slutzky