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| ARE/ECN 115A  | Winter 2020 |
| ***GROWTH, ACCUMULATION AND CONVERGENCE******Due: Tuesday, January 28th by 1:30 PM (via Gradescope)*** |

**Instructions:**

The objective of this problem set is to become familiar with the mechanics and the implications of the Harrod-Domar and Solow growth models. An excel template to this problem set can be found in the Problem Set 1 folder on Canvas. You should fill out the excel and copy each of the figures and paste them in the designated boxes in this document. For each graph, make sure to provide the chart title, legend (if applicable) and label the axes. Feel free to work individually or in small groups (<=3). **Your group will turn in one PDF of this document on Gradescope. When submitting, you will need to create a group on Gradescope and include all your group members’ names in addition to listing them on this document. Please fill in the questions below in their designated boxes**. **Please provide final equations along with numeric answers wherever necessary. DO NOT change the size of the boxes. Also, paste the graphs as images into this word file.**

**1. Income per Capita, Well-being and Growth: World Bank Data**

A. As we argued in class, income per-capita is a good yardstick of development if it is strongly correlated with important indicators that directly measure welfare, such as health and gender equality. In this problem, you will empirically explore the strength of the relationship between income per capita and several indicators of women’s empowerment. To answer this question, download the Excel file “ARE 115A WQ20 PS1.xls” on Canvas. Refer to the sheet named “World Bank data” for this problem. This file contains 2015 data for 161[[1]](#footnote-1) countries and comes from the World Bank’s country-level data sets publicly available at: <http://data.worldbank.org/indicator>. The variables of interest are as follows:

* **y**: GDP per capita in constant 2010 US$.
* **Parliament:** % of parliamentary seats in a single or lower chamber held by women
* **Labor Ratio:** Ratio of women in the labor force to men in the labor in the labor force
* First, complete the spread sheet by computing the Labor Ratio. What is Labor Ratio of Rwanda and Spain? [Round your answer to 3 decimal places]

Rwanda:

Spain:

* Second, generate the following scatter plots: 1) Parliament versus *y* (title it “Figure 1A”),and 2) Labor Ratio versus *y (*title it “Figure 1B”). In each plot, put the empowerment variable on the vertical axis and *y* on the horizontal axis and add a trendline through the data points.

Figure 1B

Figure 1A

B. Interpretation: What features stand out to you? Does higher income per capita seem to lead to greater empowerment of women?

C. Now, let’s look at a real-world example of economic growth. We want to see the rate at which Peru’s per capita income grew between 1990 and 2018. To answer this question, go to the World Bank country-level data portal above. Find the indicator GDP per capita (constant 2010 US$). Once you select Peru, make sure you set the start date at 1990 and the end date at 2018. Assuming a constant growth rate over this period, what rate has Peru grown at? [*HINT: use the constant growth rate formula from lecture and discussion:* $y\left(t\right)=y\left(0\right)\left[1+g^{\*}\right]^{t}$]

**2. Mechanics of the Harrod-Domar Growth Model**

Now let’s walk through the mechanics of the Harrod-Domar model for Wakanda Kingdom. Let’s assume the following parameter values for Wakanda: The incremental capital-output ratio, *v*, is 1.5; capital depreciates at a rate of 2% per year (*d* = 0.02); the population grows at 2% per year (*n* = 0.02); and the savings rate is 10% (*s* = 0.1).

A. In the initial year (*t=0*) the per-capita capital stock equals 100 and the population is 1(measured in millions). Assuming the economy evolves according to the Harrod-Domar model, fill in Table 2 below. (NOTE: First, completely fill in columns A – G using the production function from the H-D model and the equation for capital accumulation, then calculate the growth rates using the definition of growth rate: *g(t) = [Y(t+1)-Y(t)]/Y(t))*

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| **Table 2. Evolution of a Harrod-Domar Economy** |
| (A)Yeart | (B)Popu-lationL(t) | (C)TotalCapital StockK(t) | (D)Per CapitaCapital Stockk(t) | (E)Total IncomeY(t) | (F)Per CapitaIncomey(t) | (G)Total SavingsS(t) | (H)Growth Rate of Total Incomeg(t) | (I)Growth Rate of Per Capita Incomeg\*(t) |
| 0 | 1.00 | 100.00 | 100.00 |  |  |  |  |  |
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  | NA | NA |

For the rest of this problem, you should set up a spreadsheet with the same structure as Table 2 above. To do so, create a new sheet in the Excel file (ARE 115A WQ19 PS1) and name it “Harrod-Domar”

B. Using the same parameter values, fill out through year 100. What is Wakanda’s income per capita in years 50 and 100?

Year 50:

Year 100:

jhkjhkjhkjhmnljknk,

C. Our second kingdom is called *Asgard*. *Asgard* is exactly the same as *Wakanda* except that it starts with a greater capital stock. In the initial year (*t=0*), *Asgard* has a capital stock of 300. On the same spread sheet, create a similar table for Asgard and fill out through year 100.

* What is Asgard’s income per capita in years 50 and 100?

Year 50:

Year 100:

* Now, on a single figure, graph income per capita for both Wakanda and Asgard from year 0 to year 100. Title your chart “Figure 2B/C: Per Capita Income Paths in Harrod-Domar”

Figure 2B/C

* Do Wakanda and Asgard converge over time given their common savings rate according to this model? [YES or NO]

D. Finally, suppose Killmonger won the “ritual combat” and the Blank Panther was indeed dead. Killmonger realizes that most of the Wakandans’ loyalty still lies with the King Tchaka’s family. To win over people’s hearts, he adopts populist policies, such as, growing the economy to catch Asgard, a longtime rival of Wakanda. Killmonger is a handsome and brilliant orator and is confident that he can convince the people of Wakanda to consume less and save more in order to fulfill his economic catch-up mission. Having grown up in Oakland, he is aware of the UC Davis Managerial Econ program’s reputation. So, he hires you to tell him what savings rate is necessary to catch up with Asgard over a 50-year time horizon.

* If both kingdoms grow at a constant rate, what rate of per capita income growth ($g^{\*}$) will *Wakanda* need to catch up to *Asgard* over this time horizon? [Hint: *use the constant growth rate formula – as in 1C -- to answer this question*]
* What is the savings rate? *[Hint: use the approximate formula for growth rate of income per capita in the H-D model to answer this question.]*

* Now on the same figure, plot per capita income for Asgard which you computed using the saving rate in C above and for Wakada using the old savings rate (as in B above) and the new savings rate you just computed above (Figure 2D). Title chart “Figure 2D: Per capita paths in the Harrod-Domar model” [*These are three line graphs: one for Asgard and 2 for Wakanda*]

Figure 2D

* By how much is Wakanda’s GDP under the new savings rate greater than that under the old savings rate in year 100? [*no math required, read the values from the figure and use a calculator to compute the difference*]

* E. Reflection: Do you think that Venezuela could really catch up with USA by following this savings plan? (BRIEFLY explain the rationale for your answer to this question.)

**3. Solow Model with No Technological Change**

Let’s now modify the technology assumption used in the prior problem and assume that output in Wakanda and Asgard Kingdoms is produced according to the following constant-returns-to-scale production function that lies at the heart of the standard Solow growth model:

$$Y\left(t\right)=2\left[K\left(t\right)\right]^{0.75}\left[L\left(t\right)\right]^{0.25}$$

Assume we have the same values as in problem two for the following exogenous parameters: *s = 0.10*; *n = 0.02*; *d = .02*. [*Hint: for both 3A and 3B, you should use the equation* $∆k\left(t\right)=0$ *in the Solow model*]

A. What is the steady state level of capital per worker?

B. What is the steady state level of income per worker?

Assume also that our two kingdoms start with the same initial conditions as in problem two; namely, L*(0)=1* for both kingdoms, while that *K(0) = 100* in *Wakanda* and *K(0) = 300* in *Asgard*.

For parts C – D, you will need to create graphs in Excel. You should set up a spreadsheet with a similar structure as Table 2 above. To do so, create a new sheet in the same file as above and name it “Solow no tech”

C. On one graph plot the income per capita levels, y,for the two kingdoms over 200 years under no exogenous technological change. Title this graph “Figure 3C: Income Paths of Wakanda and Asgard under Solow I”.

Figure 3C

D. On a separate graph, plot the income per-capita growth rates over 200years for both countries. Title this graph “Figure 3D: Growth Rates of Income Per Capita under Solow I”.

Figure 3D

E. Based on your graphs, does *Wakanda* seem to be converging to *Asgard* in terms of $y$ and $g^{\*}$? [YES OR NO]

F. Reflection: How and why are the implications of this model for *Wakanda* different from those in the Harrod-Domar model? Why can growth not be sustained?

G. Suppose now that Killmonger raises the savings rate in *Wakanda* to the level that you identified in 2D above.

* What happens in both short and long terms to per-capita income levels, y, and growth rate, $g^{\*}$, after the savings rate increases in *Wakanda*? [A short answer will suffice]
* Do *Asgard and Wakanda* still converge in the long run? Why or why not? [A short answer (3 lines max) will suffice]

 **4. Solow Model with Exogenous Technological Progress**

Now assume that there is an exogenous rate of technological progress of 1% per year. To capture this, we need to slightly modify the production function as follows:

$$Y\left(t\right)=2\left[K\left(t\right)\right]^{.75}\left[T\left(t\right)L\left(t\right)\right]^{.25}$$

In the equation above, $T\left(t\right)$, is the state of technology and measures how “effective” our workers are. To capture exogenous technological progress, assume that in our initial period, $T\left(0\right)=10$ and that $T\left(t\right)$ grows exogenously by 1% every year for every kingdom.

Assume that both *Wakanda* and *Asgard* have a 10% savings rate and that initial capital stocks and all other exogenous parameters are the same as in problem 3.

A. What are the steady state levels of capital per *effective* worker and income per *effective* worker?

B. For this part, you will need to create graphs in Excel. You should set up a spreadsheet with a similar structure as in Question 2 and 3 above. To do so, add a new sheet to the same file as above and name it “Solow with Tech”. In contrast to Question 3, be sure to add a column to your Excel spreadsheet to track the state of technology, $T\left(t\right)$.

* On one graph plot the per capita income levels, y, for the two kingdoms over 300years under exogenous technological change. Title this graph “Figure 4A: Income Paths under Solow with Exogenous Technical Change”.

Figure 4A

* On a new graph, plot the per-capita income ***growth rates,*** $g^{\*}$, over 300years for both kingdoms. Title this graph “Figure 4B: Growth Rates in Solow Model with exogenous Technical change”.

Figure 4B

C. Reflection: Compare and contrast your findings in 4A with the results you obtained under the Harrod-Domar model. Even though long run *growth rates* converge between *Asgard* and *Wakanda* in both the Harrod-Domar and this “Solow-with-exogenous-technological-change” model, why do per-capita income *levels* converge in one model, but not in the other?

**THE END**[[2]](#footnote-2)

1. The World Bank data set contains information on over 220 countries. I have included only those countries with complete data for all the variables in 2015 and I also deleted countries with per capita income greater than 50,000. [↑](#footnote-ref-1)
2. The author is aware of any fictional characters used in this PS. [↑](#footnote-ref-2)