Enhancing Mathematical Learning in a Technology-Rich Environment

In *Principles and Standards for School Mathematics* (NCTM 2000), the Technology Principle asserts: "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24). More specifically, a technology-rich environment for mathematical learning influences five critical features of the classroom (Hiebert et al. 1997): the nature of classroom tasks, the mathematical tool as learning support, the role of the teacher, the social culture of the classroom, and equity and accessibility. An essential question when working in a technology-rich mathematics environment is how technology can be used (appropriately) to enhance the teaching and learning of mathematics.

This article describes teachers working collaboratively in a technology-rich environment to plan mathematics lessons that address the needs of their diverse students, in particular, English Language Learners (ELLs) and students with special needs. Through classroom examples, we discuss how a technology-rich learning environment influences a classroom's critical features. Moreover, we define

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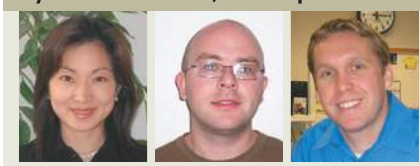

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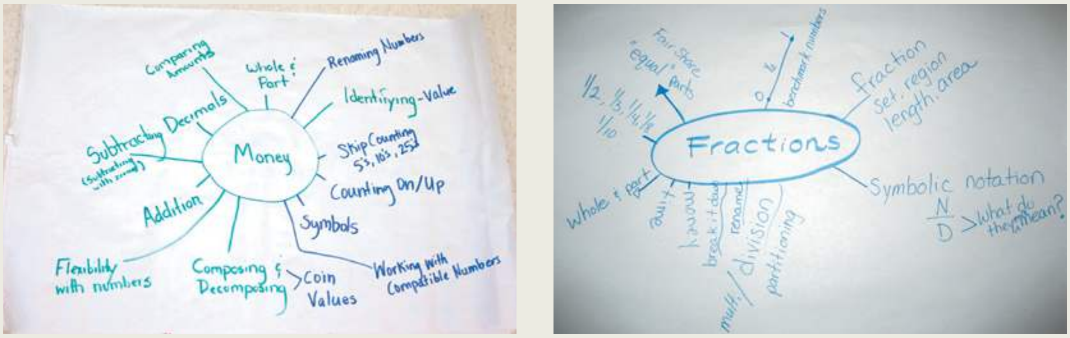
Figure 1

Student work showing mathematical knowledge mapping



unique technological properties that amplify opportunities for extending mathematical thinking.

The participating school is a Title I elementary school in a major metropolitan area with approximately 600 students: 51 percent Hispanics, 24 percent Asians, 16 percent Caucasians, 3 percent African Americans, and 6 percent others. More than 50 percent of the student population receive free or reduced lunch, 44 percent receive services for English for Speakers of Other Languages, and 49 percent are identified as limited in English proficiency. On the basis of student need at this school, an important school initiative sought to incorporate nonlinguistic representations into students' daily activities to help build their background knowledge. Representing knowledge (nonlinguistic representation) is one of nine categories of instructional strategies proven to advance student achievement (Marzano et al. 2001). An essential part of the initiative promotes the integration of technology in all content areas to provide the diverse student population with interactive, visual, and multimedia tools. To enact the initiative, teachers in each grade level collaborated with a university mathematics educator to design lessons incorporating technology tools and nonlinguistic representations to engage, motivate, and respond to the needs of a diverse group of learners.

To begin planning collaborative lessons, teachers identified a mathematics area at their grade level that presented a teaching and learning challenge. For one of the highlighted classroom examples, we will share a third-grade money lesson that was part of the measurement strand. The lesson objective was to count a collection of mixed coins and then find and record a variety of ways to show a given amount of money. The future building-block target was to make change for amounts up to five dollars.

The second featured lesson was a fourth-grade fractions lesson with the objective of renaming fractions and finding equivalent fractions. This lesson was a prerequisite to adding and subtracting with unlike denominators using models. Once teachers identified the lesson objectives, the lesson-planning team worked together to construct a mathematics knowledge map outlining the key components of both interrelated prerequisite and future knowledge mathematics concepts building blocks. Additionally, they identified effective representations or models to teach each concept (see **fig. 1**).

In these two lessons, the planning team included third- and fourth-grade classroom teachers, the special education teacher assigned to those grade levels, the mathematics specialists, and the university mathematics educator. Teachers ranged from novices to experienced teachers with varied strengths and weaknesses in the areas of technology integration, mathematical content knowledge, and teaching practice—a range that provided opportunities for all participants to develop deeper pedagogical content or technology knowledge.

Creating Technology-Rich Mathematics Learning Environments

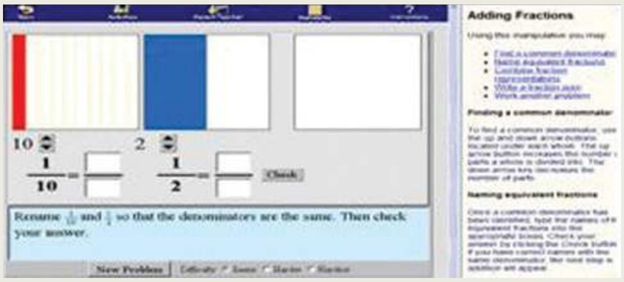
When creating a technology-rich mathematical learning environment, teachers must understand what using technology "appropriately" (Garofalo et al. 2000, p. 67) means when integrated into teaching mathematics:

1. Introducing technology in context
2. Addressing worthwhile mathematics with appropriate pedagogy
3. Taking advantage of technology

Figure 2

An Advancing Mathematical Thinking planning sheet

Advancing Mathematics Thinking with the Use of Technology

<table>
<tr><td>Web site</td><td colspan="2">Addition of Fractions</td><td></td></tr>
<tr><td>Math Strand</td><td>Number and Operations</td><td>Grade Level</td><td>Fourth Grade</td></tr>
<tr><td>Description of mathematical concept (NCTM)</td><td colspan="3">

</td></tr>
<tr><td colspan="3">Analysis of Mathematical Representations and Models</td><td></td></tr>
<tr><td colspan="4">X Concept tutorial/skill practice ______ Investigation/problem solving _____ Open exploration</td></tr>
<tr><td colspan="3">Representation
• Create and use representations to organize, record, and communicate mathematical ideas
• Select, apply, and translate among mathematical representations to solve patterns
• Use representations to model and interpret physical, social, and mathematical phenomena</td><td>Connected pictorial and numerical representations</td></tr>
<tr><td colspan="3">Communication
• Organize and consolidate their mathematical thinking through communication
• Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
• Analyze and evaluate the mathematical thinking and strategies of others
• Use the language of mathematics to express mathematical ideas precisely</td><td>Peer talk:
Discuss applet's function and the mathematics process (step by step)</td></tr>
<tr><td colspan="3">Connections
• Recognize and use connections among mathematical ideas
• Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
• Recognize and apply mathematics in contexts outside of mathematics</td><td>Connecting renaming before combining; finding common multiples with arrows breaking pieces</td></tr>
<tr><td colspan="3">Reasoning and Proof
• Recognize reasoning and proof as fundamental aspects of mathematics
• Make and investigate mathematical conjectures
• Develop and evaluate mathematical arguments and proofs
• Select and use various types of reasoning and methods of proof</td><td>Analyzing and making sense of the algorithmic process</td></tr>
<tr><td colspan="3">Problem Solving
• Build new mathematical knowledge through problem solving
• Solve problems that arise in mathematics and in other contexts
• Apply and adapt a variety of appropriate strategies to solve problems
• Monitor and reflect on the process of mathematical problem solving</td><td>Discovering what happens when fraction pieces are renamed and combined</td></tr>
</table>

4. Connecting mathematics topics
5. Incorporating multiple representations

In planning the lessons, we used these guidelines to structure the learning environments with virtual manipulatives and applets.

In addition to knowing how to integrate technology appropriately, teachers must focus on worthwhile mathematics and effective pedagogy when using technology. An effective way to optimize the mathematical thinking opportunities presented by technology is to plan the mathematics task focused on the five Process Standards (NCTM 2000): Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. We used a template during planning to guide the activity and the classroom discourse so that teachers were focused on advancing students' mathematical thinking processes (see **fig. 2**).

The remainder of the article describes two lessons in which technology was used as an instructional strategy. More specifically, we describe the task and the technological tool that supported the learning, the role of the teacher in capitalizing on learning in the technology-rich environment, and how the technology gave more access to learning opportunities and more equity to diverse learners.

Counting Change Makes Sense

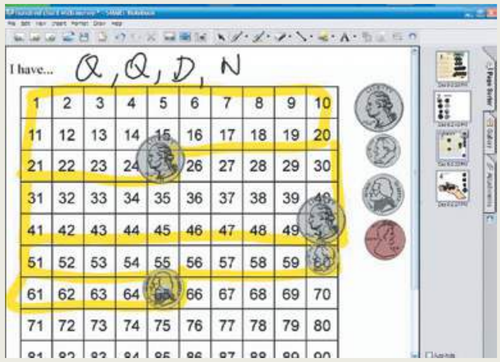
The third-grade lesson objectives were to teach students to count a collection of mixed coins and find and record a variety of ways to show a given amount of money. Making change for amounts to five dollars was a future knowledge building block. To address the objectives, we designed the task on the SMART Board with a hundreds chart and coins that had infinite clones to count change. Using the hundreds chart (see **fig. 3a**), students worked with benchmark numbers such as five, ten, and twenty-five, learned to skip-count when counting change, and practiced using numbers flexibly. The second activity, "Show Me the Money," embedded two tasks. First, students counted the money in the virtual hand (see **fig. 3b**) by dragging the coins and skip-counting. The following scenario offered the other task: "I have in my hand a total of thirty-three cents. Show me all the possible ways to make that amount" (see **fig. 3c**).

We used technology to provide students with multiple representations. The electronic hundreds chart helped students see the relationship between coins and their value. Using the SMART Board, students were able to touch the screen and drag the coin directly onto the hundreds chart to help them count by twenty-five, ten, five, or one. We used the highlighting pen to shade in money amounts and to show the value of each coin. For many children, counting money is especially challenging because its representation is nonproportional; that is, although a dime has more value than a nickel, the dime is

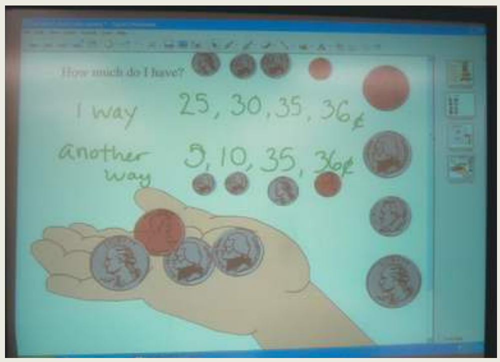
Figure 3

Student work using tech tools

(a) The SMART Board to count change



(b) A virtual hand to drag coins and skip-count



(c) Multiple coin combinations for thirty-three cents

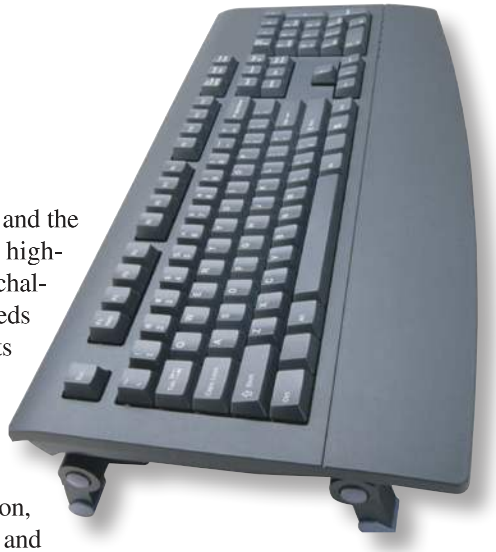
physically smaller. The task was designed to relate the proportional representation of the hundreds chart with the nonproportional representations of the coins so that students would gain better understanding of each coin's magnitude and worth. Demonstrating how to count up to thirty-three cents at the SMART Board, one student commented, "With twenty-five cents, I need to shade in a lot more: two rows for the two tens and five ones; and to get to thirty cents, I need to add a nickel and then three more pennies." Shading the hundreds chart was an instructive visual representation of the coin values.

The teacher's role

The SMART Board technology facilitated the teacher's ability to give students opportunities to show multiple ways to count change. Important teacher tactics included allowing students to display different solution paths on the SMART Board simultaneously, asking students to compare different thinking strategies for making compatible numbers, and initiating productive discussion on efficient change-counting strategies. Simultaneously displaying multiple student solutions allowed students to compare and make some important generalizations about counting coins. For example, when given the coins (a quarter, dime, dime, and nickel), one student shared, "I count the quarter first and then the nickel to get to thirty cents [and] then add the two dimes to get to fifty cents." Another student said, "It is easier for me to add the quarter, then the two dimes to go from twenty-five, thirty-five, forty-five cents, [and] then add the nickel to get to fifty cents." Many students began to adapt their thinking and model the strategies shared in class that made it easier to skip-count money. The task also allowed them to discover ways to compose and decompose numbers using different coin combinations.

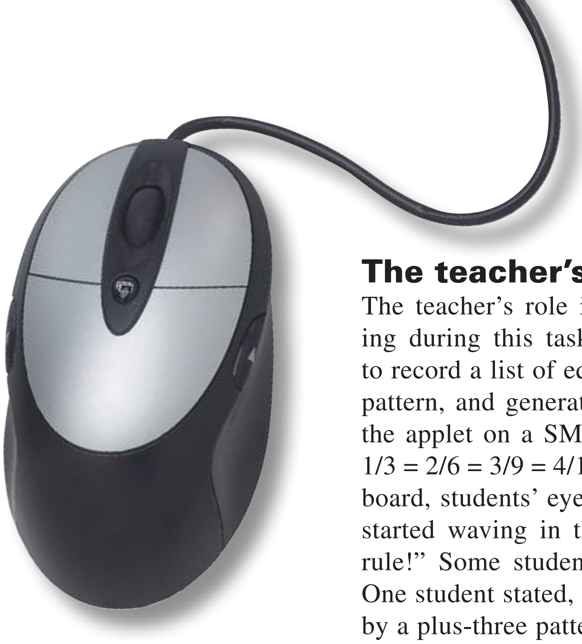
Equity and access for diverse learners

Technology enhanced students' learning by allowing diverse learners to understand the concept through multiple representations. Students recorded the numeric value right next to the coins as they counted change on the hundreds chart, thereby allowing the visual representations to be closely tied to the numeric representations. For some English Language Learners, being able to write words such as *quarter*, *dime*, *nickel*, and *penny* next to the coin gave them better access to the lesson. The technology features allowed for better communication, problem solving, reasoning, and connections among concepts. In fact, the dual representations of the coins and the hundreds chart allowed for some high-ability students to engage in more challenging tasks. By using the hundreds chart and counting on, these students used the tools to determine how much change one should get back if one pays with a dollar bill. For example, the cost of a candy bar is sixty-eight cents. The child counts on, "Sixty-nine, seventy," using pennies and then counts on, "Eighty, ninety, one hundred," using three dimes; the total is thirty-two cents in change. Having multiple tasks embedded within each task also allowed for differentiation in instruction.



Exploring Equivalent Fractions

The lesson objectives for the fourth-grade fractions lesson were to rename fractions and find equivalent fractions; the subsequent lesson focused on using models to add and subtract fractions with unlike denominators. The virtual manipulatives called Fraction Equivalence, found on the National Library of Virtual Manipulatives Web site, allowed students to explore the relationship between equivalent fractions. On the Fraction Equivalence applet, students were presented with a partially shaded circle or square and the fraction symbol. They were directed to "find a new name for this fraction by using the arrow buttons to set the number of pieces. Enter the new name and check your answer." To do this, students clicked on arrow buttons below the whole unit, which changed the number of parts. When students had an equivalent fraction, all lines turned red. When a common denominator was identified, students typed the names of the equivalent fractions into the appropriate boxes. They checked their answers by clicking the "Check" button. Each step of the way, the pictures were linked to numeric symbols that dynamically changed with the students' moves (see **fig. 4a**). To help explore the relationship between equivalent fractions, the applet prompted students to find several equivalent fractions. This applet was specifically designed to develop the concept of renaming fractions. Although constrained to one specific objective, the tool allowed for more exploration than do physical manipulatives, such as fraction circles or bars, which are usually limited by the number of fractional pieces. This applet allowed students to equally divide a whole, up to ninety-nine pieces, and generate multiple equivalent fraction names.



The teacher's role

The teacher's role in extending students' thinking during this task was to encourage students to record a list of equivalent fractions, look for a pattern, and generate a rule. For instance, using the applet on a SMART Board, a student found 1/3 = 2/6 = 3/9 = 4/12. As we recorded this on the board, students' eyes started to widen and hands started waving in the air: "Oh, oh, I know the rule!" Some students noticed the additive rule. One student stated, "The denominators are going by a plus-three pattern." Another student echoed, "It is like skip counting." And another voiced, "It is the multiples of three."

To get students to further explore the relationship, the teacher asked them to examine the multiplicative pattern for both the numerator and the denominator in 2/3. Students listed 2/3 = 4/6 = 6/9, and again they quickly saw the additive pattern and the multiples of two for the numerator and three for the denominator. Then the teacher posed the questions, "Are 2/3 and 20/30 equivalent fractions? What about 2/3 and 10/15?" To find a rule beyond the additive rule, students were asked to use the applet and talk to their partners while exploring the relationships between the equivalent fractions and to other fractions. When students came back together as a group, several of them shared their discoveries: "The fractions 2/3 and 20/30 are equivalent, because you multiply both numerator and denominator by ten. And in 2/3 = 10/15, you multiply both numerator and denominator by five." These comments led to a lively conversation about how 10/10 and 5/5 both equal one whole. The teacher connected this idea to the identity property of multiplication by asking, "What happens when we multiply one by any number?" The ensuing discussion reinforced the idea that no matter how you rename the fractions, as long as you multiply them by one or *n*/*n*, you will have an equivalent fraction. To challenge the students, the teacher posed a question: "What would the equivalent fraction be for 1/3 if the denominator were divided into ninety-nine parts?" This type of questioning encouraged students to extend their thinking by making conjectures and testing their rule or hypothesis.

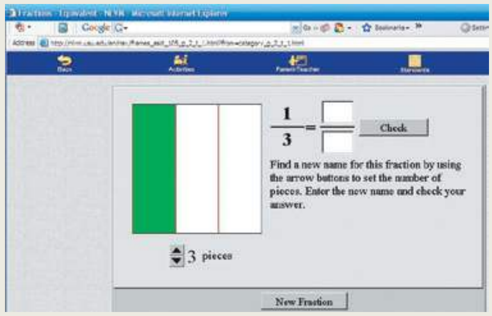
Equity and access for diverse learners

Instead of merely teaching an algorithm, we used the fraction applet to allow all the students to think and reason about the relationships among

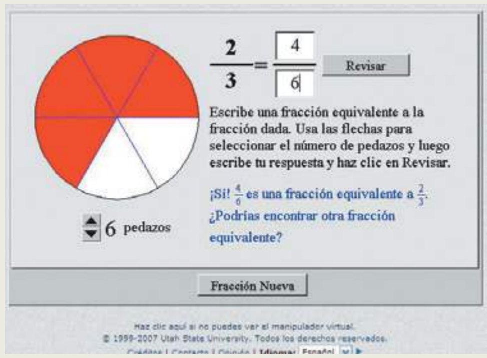
Figure 4

Fraction equivalence applet

(a) in English



(b) in Spanish



the opportunity to work with a partner. As the pairs worked together with the applet, they were able to make sense of the mathematics by talking through the processes. The teacher paired limited English-proficient students with students who spoke the same language and could better explain what was happening. The ability to switch to Spanish gave many ELLs better access to the mathematics (see **fig. 4b**). And finally, while other students explored with a partner, the special needs learners worked together in a small group with the mathematics educator, who scaffolded their experience by working collaboratively in front of the SMART Board.

Traditionally, special needs learners are often given direct instruction on how to perform an algorithm using mnemonic devices or procedural steps without being given opportunities to construct conceptual understanding of the procedure. One of the biggest challenges of working with physical manipulatives, such as fractions circles, is that actually manipulating multiple pieces creates so much of a cognitive load on students' thinking processes that they lose sight of the mathematical concept. In our classroom example, working with a virtual applet reduced some of the physical manipulation so that the special needs students could focus more on mathematical processes and relationships among the equivalent fractions. In many ways, the applet gave special needs students access to the mathematics without creating a cognitive overload.

Having visual and numeric representations closely tied together and displayed on the screen helped students make direct connections in relationships among equivalent fractions. Throughout the lesson, the teacher worked with a small group of ELLs and special needs students, who required more teacher support and benefited from small-group interaction. The teacher could re-teach and reinforce skills as needed. The kinesthetic and tactile advantages of the SMART Board also enabled students to grasp greater understanding of the concept as they took turns manipulating the SMART Board and coaching each other through the given task.

Leveraging Technology to Enhance Mathematical Learning

Learning environments that take advantage of virtual manipulatives and applets offer a number of ways for students to develop their mathematical understanding. The authors identify the following as five primary benefits of virtual manipulatives and applets:

1. Linked representations provide connections and visualization between numeric and visual representations.
2. Immediate feedback allows students to check their understanding throughout the learning process, which prevents misconceptions.
3. Interactive and dynamic objects move a noun (mathematics) to a verb (mathematize).
4. Virtual manipulatives and applets offer opportunities to teach and represent mathematical ideas in nontraditional ways.
5. Meeting diverse learners' needs is easier than with traditional methods.

Leveraging Technology in Mathematical Teaching and Learning

As teachers structure their learning environments using technology, the primary focus should be to support mathematical understanding. A number of design and assessment issues are unique to using technology. For example, teachers should consider having students print their work or use a task sheet to record their work, their thoughts, and examples from using the virtual manipulative or applet. By writing and recording their work, students reflect on their own thinking, a metacognitive process, which is essential in problem solving. The task sheet also provides a permanent record that can be used for the teacher's assessment purposes.

Ensuring mathematical discourse with peers and teachers before, during, and after using a technology tool is an important design issue, critical to students' exploration of patterns and relationships. Using appropriate technology in teaching and learning should make learning environments qualitatively different from teaching without technology. That is, integrating technology should not merely add a virtual representation to a lesson; it should enhance teaching and learning by providing opportunities for rich mathematical thinking and discussion. Teachers should consider specific pedagogical issues. In our two classroom examples, we illustrated how using the NCTM Process Standards alongside the unique aspects of the technology tools allowed meaningful learning to take place while meeting the needs of diverse learners.

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