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Research on the supplier selection model of closed-loop logistics systems with hesitant fuzzy information[1]

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Abstract. With the legislation is perfected and the useful resources decreased day by day, there are more and more enterprises are interested in closed-loop logistic systems, especially about remanufacturing logistics and reusing logistics. As we know, location, routing, and inventory are the most important factors in logistic systems, and there are closed relationships among them. In this paper, we study on the multiple attribute decision making with hesitant fuzzy information. Inspired from the idea of induced OWG (IOWG) operator, we propose the induced hesitant fuzzy Hamacher ordered weighted geometric (IHFHOWG) operator and then utilize IHFHOWG operator to develop a novel method for multiple attribute decision making with hesitant fuzzy information. In the end, an illustrative example for supplier selection is proposed to test the developed method and to show the effectiveness of the proposed algorithm.

Keywords: Multiple attribute decision making, hesitant fuzzy information, induced hesitant fuzzy Hamacher ordered weighted geometric (IHFHOWG) operator, induced OWG operator, supplier selection

1. Introduction

Voluntary Inter-industry Commerce Standards (VICS) in North America introduced Collaborative Planning, Forecasting and Replenishment (CPFR), exception treatment, multi-level collaboration and synchronization. CPFR can be seen as a requirement-oriented, improved supply chain, proposing a working flow set. The proposed working flow is designed based on the improvement of consumers' value as shared objectives, using collaboration of supply-chain enterprises, the share of standardized information, construct objective plans, the practice of market forecasting, effective production and inventory management, together with timely replenishment according to dynamic requirements. Therefore, performance and efficiency of the whole supply chain is able to be promoted, demonstrating the thought of collaborative management of the supply chain. The notion and methods of CPFR have many advantages, that can be widely used in supply-chain enterprises. The leading enterprises are inspired by using CPFR. But, CPFR and the related working flow can be acceptable rather than being put into practice, and various devices and approaches, involved in the working flow, are to be tackled. The related studies all over the world demonstrates that theory and method of supply-chain collaboration are mainly concerned with theory, thought, system and simulation at strategic and tactic levels, with quantitative analysis mainly containing supply-chain inventory collaboration. This includes empirical analyses and simulation experiments. In

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their findings, we analyze the overall collaboration of leading enterprises with their forward and backward enterprises and multi-hierarchical analyses of supplier selection in the environment of collaboration, nor analyses of the role of factors in sales forecasting and demand information and their application in supply-chain enterprises. The Information Theory has advanced, and it can be exploited to many natural and social disciplines. Nevertheless, its relationship using the managerial rule, together with the possible representation in the economic information and managerial conduct, is to be investigated. Furthermore, there are only a few of studies which focused on the supply-chain collaboration. Meanwhile, the representations of information in supply-chain collaboration are not very important in the existing works. Studies on these gaps are able to produce information and its representation forms in supply-chain collaboration, delineating an outline and direction. Suppose that this is compared and analyzed according to the physical information and its features actually, it is possible that the study on supply-chain collaboration can be enhanced to a philosophical level.

Supply chain has been a complicated network instead of former simple chain, with the span-new environment and boundless opportunities supplied by the prosperous e-business, so that the companion selection has become more and more intricate. As the start of supply chain, vendors are responsible for the input of resources and related with a lot of fields in trades. Therefore, rational vendor selection is the key segment in manufacture. Enterprises should evaluate all the potential vendors and then select some of them according to the results of comparison and actuality so as to build a stable strategic alliance and to improve their competitiveness. That's the foundation of supply chain. However, restrained by all kinds of factors, our nation has no normative or perfect system for vendor selection. In the face of hundreds of suppliers, the enterprises usually emphasize particularly on cost and choose the cheapest one from so many options. Such a short-term action will affect the relation between both sides and result in the decline of quality and the delay of delivery and so on. In the new economic integration situation of the customer demands is personalized and the competition is increasingly fierce, the enterprise competence depends on wether it can make use of other rescorce effectively. Numerous enterprises have changed the role as the interest community to keep strategic partnership with others. In the environment of supply chain, the enterprise depends on customer demands, seeks the strategic supplier parters by scientific method of evaluation and selection rapidly, and implements effective management, the target of the progress is to hoisting its core capacity and obtain competitive advantage. We concentrate on the multiple attribute decision making problem with hesitant fuzzy information [1–12]. Using the induced OWG (IOWG) operator, we propose the induced hesitant fuzzy Hamacher ordered weighted geometric (IHFHOWG) operator and then utilize IHFHOWG operator to present a method for multiple attribute decision making with hesitant fuzzy information. In the end, an example for supplier choosing is designed to test the effectiveness of our proposed method.

2. Preliminaries

Atanassov [13, 14] proposed the definition of intuitionistic fuzzy set(IFS) based on the concept of fuzzy set [15]. The intuitionistic fuzzy set has obtained many attentions for its high quality performance [16–18]. Moreover, Torra et al. [19] proposed a hesitant fuzzy set that allows the membership with several possible values and studied on the relationship between hesitant fuzzy set and intuitionistic fuzzy set, and demonstrated that the envelope of hesitant fuzzy set is an intuitionistic fuzzy set. Xia and Xu [20] proposed some hesitant fuzzy information aggregation operators and their application to multiple attribute decision making. Xu and Xia [21, 22] developed the distance and correlation measures with hesitant fuzzy information. For more studies with hesitant fuzzy information, please refer to references [24–35].

Next, several concepts of hesitant fuzzy sets are given as follows.

Definition 1. [19] Assume that there is a fixed set X, then a hesitant fuzzy set (HFS) on X is according to a function that when utilized to X returns a subset of [0, 1]. Afterwards, Xia and Xu [21] represent the HFS by the following equation as follows.

$$E = (\langle x, h_E(x)\rangle \,|x \in X), \tag{1}$$

where $h_E(x)$ refers to some values in [0, 1], which means the membership degree of the element $x \in X$ to the set E. Thus, Xia and Xu et al. [20] define $h = h_E(x)$ as a hesitant fuzzy element(HFE).

Definition 2. [20] Gvien a hesitant fuzzy element h, $s(h) = \frac{1}{\#h}\sum_{\gamma\in h}\gamma$ is named the score function of h, where $\#h$ refers to the number of the elements in

h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Using the correlation between the HFEs and IFVs, Xia et al. [20] propose several operations on the HFEs h, h_1 and h_2:

(1) $h^\lambda = \cup_{\gamma \in h} \left\{ \gamma^\lambda \right\}$;

(2) $\lambda h = \cup_{\gamma \in h} \left\{ 1 - (1-\gamma)^\lambda \right\}$;

(3) $h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}$;

(4) $h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1\gamma_2\}$.

T-norm and t-conorm is belonged to fuzzy set theory, that are utilized to describe a generalized union and intersection of fuzzy sets [21]. Roychowdhury et al. [22] proposed a definition of t-norm and t-conorm. Exploiting a t-norm (T) and t-conorm (T^*), a generalized union and a generalized intersection of intuitionistic fuzzy sets are discussed by Deschrijver et al. [23]. Next, Hamacher [24] designed a more generalized t-norm and t-conorm.

Hamacher product $\otimes$ is a t-norm and Hamacher sum $\oplus$ is a t-conorm, where

$$T(a,b) = a \otimes b = \frac{ab}{\gamma + (1-\gamma)(a+b-ab)}$$

$$T*(a,b) = a \oplus b = \frac{a+b-ab-(1-\gamma)ab}{1-(1-\gamma)ab}$$

Especially, when $\gamma = 1$, then Hamacher t-norm and t-conorm can reduce to

$$T(a,b) = a \otimes b = ab$$

$$T*(a,b) = a \oplus b = a+b-ab$$

which denote the algebraic t-norm and t-conorm, where $\gamma = 2$, then Hamacher t-norm and t-conorm can be reduced to

$$T(a,b) = a \otimes b = \frac{ab}{1+(1-a)(1-b)}$$

$$T*(a,b) = a \oplus b = \frac{a+b}{1+ab}$$

which are named as Einstein t-norm and t-conorm respectively.

Inspired by the Hamacher aggregation operators, the product $\otimes$ and the Hamacher sum $\oplus$, afterwards, generalized intersection and union on two HFEs h_1 and h_2 become the Hamacher product(denoted by $h_1 \otimes h_2$) and Hamacher sum(denoted by $h_1 \oplus h_2$) of two HFEs h_1 and h_2.

(1) $h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2}$
$$\left\{ \frac{\gamma_1 + \gamma_2 - \gamma_1\gamma_2 - (1-\gamma)\gamma_1\gamma_2}{1-(1-\gamma)\gamma_1\gamma_2} \right\};$$

(2) $h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2}$
$$\left\{ \frac{\gamma_1\gamma_2}{\gamma + (1-\gamma)(\gamma_1 + \gamma_2 - \gamma_1\gamma_2)} \right\}.$$

(3) $\lambda h_1 = \cup_{\gamma_1 \in h_1}$
$$\left\{ \frac{(1+(\gamma-1)\gamma_1)^\lambda - (1-\gamma_1)^\lambda}{(1+(\gamma-1)\gamma_1)^\lambda + (\gamma-1)(1-\gamma_1)^\lambda} \right\},\ \lambda > 0;$$

(4) $(h_1)^\lambda = \cup_{\gamma_1 \in h_1}$
$$\left\{ \frac{\gamma(\gamma_1)^\lambda}{(1+(\gamma-1)(1-\gamma_1))^\lambda + (\gamma-1)(\gamma_1)^\lambda} \right\}, \lambda > 0.$$

3. Induced hesitant fuzzy hamacher ordered weighted geometric operators

Xu et al. [25] proposed an induced OWG (IOWG) operator, which refers to an aggregation operator to utlize order inducing variables in the reordering of the arguments. Therefore, we use reordering processes to illustrate the problem with a complete mode. As is well known that IOWG operator has been successfully applied. The induced OWG operator is defined as follows.

Definition 3. [25] An IOWG operator with n dimension is defined as: $R^n \to R$, which is defined as $w = (w_1, w_2, \cdots, w_n)^T$, $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$, a set of order-inducing variables u_i:

$$IOWG(\langle u_1, a_1 \rangle, \langle u_2, a_2 \rangle, \cdots, \langle u_n, a_n \rangle) = \prod_{j=1}^{n} \left(a_{\sigma(j)} \right)^{w_j} \quad (2)$$

$a_{\sigma(j)}$ is the a_i value of the OWG pair $\langle u_i, a_i \rangle$ having the jth largest u_i ($u_i \in [0,1]$), and u_i in $\langle u_i, a_i \rangle$ denotes the order inducing variable and a_i means the argument variables.

Afterwards, we propose the induced hesitant fuzzy Hamacher ordered weighted geometric (IHFHOWG) operator that is an extension of induced ordered weighted geometric (IOWG) operator given by Xu [25].

Definition 4. Let $\langle u_j, h_j \rangle$ $(j = 1, 2, \cdots, n)$ be a collection of 2-tuples, then we define the induced hesitant fuzzy Hamacher ordered weighted geometric (IHFHOWG) operator as follows:

$$\text{IHFHOWG}_w\left(\langle u_1, h_1\rangle, \langle u_2, h_2\rangle, \cdots, \langle u_n, h_n\rangle\right) = \bigotimes_{j=1}^{n}\left(h_{\sigma(j)}\right)^{w_j} \quad (3)$$

where $w = (w_1, w_2, \cdots, w_n)^T$ is a weighting vector, such that $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$, $j = 1, 2, \cdots, n$, $h_{\sigma(j)}$ is the h_j value of the IHFHOWG pair $\langle u_i, h_i\rangle$ having the jth largest u_i $(u_i \in [0, 1])$, and u_i in $\langle u_i, h_i\rangle$ means the order inducing variable and h_i as the hesitant fuzzy arguments.

Theorem 1. *Assume that $\langle u_j, h_j\rangle$ $(j = 1, 2, \cdots, n)$ be a collection of 2-tuples, then their aggregated value by using the IHFHOWG operator is also a hesitant fuzzy variables, and*

$$\begin{aligned}&\text{IHFHOWG}_w\left(\langle u_1, h_1\rangle, \langle u_2, h_2\rangle, \cdots, \langle u_n, h_n\rangle\right)\\&= \bigotimes_{j=1}^{n}\left(h_{\sigma(j)}\right)^{w_j}\\&= \cup_{\gamma_{\sigma(1)}\in h_{\sigma(1)}, \gamma_{\sigma(2)}\in h_{\sigma(2)}, \cdots, \gamma_{\sigma(n)}\in h_{\sigma(n)}}\\&\left\{\frac{\gamma\prod_{j=1}^{n}\gamma_{\sigma(j)}^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-\gamma_{\sigma(j)}\right)\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}\gamma_{\sigma(j)}^{w_j}}\right\}\end{aligned} \quad (4)$$

where $w = (w_1, w_2, \cdots, w_n)^T$ is a weighting vector, such that $w_j > 0$, $\sum_{j=1}^{n} w_j = 1$, $j = 1, 2, \cdots, n$, $h_{\sigma(j)}$ is the h_j value of the IHFHOWG pair $\langle u_i, h_i\rangle$ having the jth largest u_i $(u_i \in [0, 1])$, and u_i in $\langle u_i, h_i\rangle$ is.

We define special cases of the IHFHOWG operator as follows.

(1) If $u_j = h_j$ for all j, then the IHFHOWG operator means a hesitant fuzzy Hamacher ordered weighted geometric (HFHOWG) operator:

$$\begin{aligned}&\text{IHFHOWG}_w\left(\langle u_1, h_1\rangle, \langle u_2, h_2\rangle, \cdots, \langle u_n, h_n\rangle\right)\\&= \bigotimes_{j=1}^{n}\left(h_{\sigma(j)}\right)^{w_j}\\&= \text{HFHOWG}_w\left(h_1, h_2, \cdots, h_n\right)\\&= \cup_{\gamma_{\sigma(1)}\in h_{\sigma(1)}, \gamma_{\sigma(2)}\in h_{\sigma(2)}, \cdots, \gamma_{\sigma(n)}\in h_{\sigma(n)}}\\&\left\{\frac{\gamma\prod_{j=1}^{n}\gamma_{\sigma(j)}^{w_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-\gamma_{\sigma(j)}\right)\right)^{w_j}+(\gamma-1)\prod_{j=1}^{n}\gamma_{\sigma(j)}^{w_j}}\right\}\end{aligned} \quad (5)$$

where $(\sigma(1), \sigma(2), \cdots, \sigma(n))$ is a permutation of $(1, 2, \cdots, n)$, such that $h_{\sigma(j-1)} \geq h_{\sigma(j)}$ for all $j = 2, \cdots, n$.

(2) If $u_j = No.j$ for all j, where j is the ordered position of the $\langle u_j, h_j\rangle$, then the IHFHOWG operator becomes the hesitant fuzzy Hamacher weighted geometric (HFHWG) operator:

$$\begin{aligned}&\text{IHFHOWG}_w\left(\langle u_1, h_1\rangle, \langle u_2, h_2\rangle, \cdots, \langle u_n, h_n\rangle\right)\\&= \bigotimes_{j=1}^{n}\left(h_{\sigma(j)}\right)^{w_j}\\&= \text{HFHWG}_\omega\left(h_1, h_2, \cdots, h_n\right)\\&= \bigotimes_{j=1}^{n}\left(h_j\right)^{\omega_j}\\&= \cup_{\gamma_1\in h_1, \gamma_2\in h_2, \cdots, \gamma_n\in h_n}\\&\left\{\frac{\gamma\prod_{j=1}^{n}\gamma_j^{\omega_j}}{\prod_{j=1}^{n}\left(1+(\gamma-1)\left(1-\gamma_j\right)\right)^{\omega_j}+(\gamma-1)\prod_{j=1}^{n}\gamma_j^{\omega_j}}\right\}\end{aligned} \quad (6)$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ be the weight vector of $\tilde{a}_j$ $(j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$.

4. Approaches to hesitant fuzzy multiple attribute decision making

Assume that $A = \{A_1, A_2, \cdots, A_m\}$ is a discrete set of alternatives and $G = \{G_1, G_2, \cdots, G_n\}$ be a set of attributes. If decision makers give several values for the alternative A_i under the state of nature G_j with anonymity. Therefore, two decision makers are able to obtain the same value. Assume that the decision matrix $H = \left(h_{ij}\right)_{m\times n}$ means the hesitant fuzzy decision matrix, in which h_{ij} $(i = 1, 2, \cdots, m, j = 1, 2, \cdots, n)$ satisfy the condition of HFEs.

Based on the above models, we design an effective approach to tackle MADM problems with hesitant fuzzy information as follows.

Step 1. Utilize the IHFHOWG operator:

$$\begin{aligned}h_i &= \text{IHFHOWG}_w\left(\langle u_{i1}, h_{i1}\rangle,\right.\\&\left.\langle u_{i2}, h_{i2}\rangle, \cdots, \langle u_{in}, h_{in}\rangle\right)\\&= \bigotimes_{j=1}^{n}\left(h_{\sigma(ij)}\right)^{w_j}\\&= \cup_{\gamma_{\sigma(i1)}\in h_{\sigma(i1)}, \gamma_{\sigma(i2)}\in h_{\sigma(i2)}, \cdots, \gamma_{\sigma(in)}\in h_{\sigma(in)}}\end{aligned} \quad (7)$$

$$\left\{ \frac{\gamma \prod_{j=1}^{n} \gamma_{\sigma(ij)}^{w_j}}{\prod_{j=1}^{n} \left(1 + (\gamma - 1)\left(1 - \gamma_{\sigma(ij)}\right)\right)^{w_j} + (\gamma - 1) \prod_{j=1}^{n} \gamma_{\sigma(ij)}^{w_j}} \right\}$$

to gain the overall preference values $h_i\,(i = 1, 2, \cdots, m)$ of the alternative A_i.

Step 2. Compute the scores $S\left(\tilde{h}_i\right)$ of the overall hesitant fuzzy preference value $\tilde{h}_i\ \ (i = 1, 2, \cdots, m)$ to rank all the alternatives $A_i\,(i = 1, 2, \cdots, m)$ and choose the optimal scheme.

Step 3. Rank all the alternatives $A_i\,(i = 1, 2, \cdots, m)$ and gain the optimal result in terms of $S\left(\tilde{h}_i\right)$ $(i = 1, 2, \cdots, m)$.

Step 4. End.

5. Numerical example

For the research field of supply chain management, individual firm cannot compete as an independent element. When building up a supply chain, supplier selection and evaluation is the most important linkage. In the long term, in order to form strategic alliance and enhance its own competitiveness and achieve a "win-win", core enterprise shall select some competent and influence suppliers. To some extent, the smooth operation and performance of the supply chain are decided by the supplier selection and evaluation. So, to evaluate and select supplier scientifically and rationally, how to establish evaluation index system and choose appropriate methods for supplier selection is a very worthy of study. Hence, in this paper, we illustrate an example for supplier choosing with hesitant fuzzy information to describe the proposed algorithm. There is a panel using 5 possible suppliers $A_i\,(i = 1, 2, 3, 4, 5)$ to choose. The experts are able to choose four attribute to evaluate the 5 possible suppliers: ①G_1 is the product quality; ②G_2 is the service; ③G_3 is the delivery; ④G_4 is the price. To prevent elements affect each other, decision makers should carefully estimate these 5 suppliers $A_i\,(i = 1, 2, 3, 4, 5)$ based on these four attributes in anonymity and the decision matrix $H = \left(h_{ij}\right)_{4\times 4}$ is given in Table 1, where $h_{ij}\,(i = 1, 2, 3, 4,\ j = 1, 2, 3, 4)$ are represented as HFEs.

We assume that the weight of IHFHOWG operator is: w = (0.30,0.10,0.40,0.20).

Table 1
Hesitant fuzzy decision matrix

	G_1	G_2	G_3	G_4
A_1	(0.5,0.7)	(0.3,0.5)	(0.7,0.9)	(0.3,0.4)
A_2	(0.5,0.6)	(0.7,0.8,0.9)	(0.2,0.3)	(0.6,0.7)
A_3	(0.7,0.8)	(0.8,0.9)	(0.3,0.5)	(0.6,0.7,0.8)
A_4	(0.6,0.7)	(0.4,0.5)	(0.3,0.4,0.5)	(0.6,0.9)
A_5	(0.3,0.5,0.7)	(0.6,0.7)	(0.4,0.6)	(0.7,0.8)

Table 2
Inducing variables

	G_1	G_2	G_3	G_4
A_1	15	13	12	9
A_2	11	20	19	22
A_3	19	12	17	13
A_4	20	13	23	24
A_5	21	15	18	13

Step 1. Experts apply order-inducing variables to represent the complex attitudinal character which contains different board directors (shown in Table 2).

Utilize the IHFHOWG operator to derive the overall preference values $h_i\,(i = 1, 2, 3, 4, 5)$ of the suppliers A_i.

Step 2. We adopt the decision information proposed in Table 1, and the IHFHOWG operator to obtain the overall values $h_i\,(i = 1, 2, 3, 4, 5)$ of the schools $A_i\,(i = 1, 2, 3, 4, 5)$ and compute the scores $s\,(h_i)\ \ (i = 1, 2, 3, 4, 5)$ of the overall hesitant fuzzy values $h_i\,(i = 1, 2, 3, 4, 5)$ of the suppliers A_i:

$$s\,(h_1) = 0.24, s\,(h_2) = 0.15$$

$$s\,(h_3) = 0.18, s\,(h_4) = 0.31$$

$$s\,(h_5) = 0.16.$$

Step 3. Rank all the suppliers according to the scores of $s\,(h_i)\ \ (i = 1, 2, 3, 4, 5)$ of the hesitant fuzzy values $h_i\,(i = 1, 2, \cdots, 5)$: $A_4 \succ A_1 \succ A_3 \succ A_5 \succ A_2$. Therefore, the supplier required is A_4.

6. Conclusion

With the legislation is perfected and the useful resources decreased day by day, there are more and more enterprises are interested in closed-loop logistic systems, especially about remanufacturing logistics and reusing logistics. As we know, location, routing, and inventory are the most important factors in logistic systems, and there are closed relationships among them. In supply chain management, individual company cannot compete as an independent entity. Supplier selection and evaluation is the most important linkage. In the long term, in order to form strategic alliance and enhance its own competitiveness and achieve a “win-win”, core enterprise shall select some competent and influence suppliers. To some extent, the smooth operation and performance of the supply chain are decided by the supplier selection and evaluation. So, in order to evaluate and select supplier scientifically and rationally, how to establish evaluation index system and choose appropriate methods for supplier selection is a very worthy of study. In this paper, we focus on the problem of multiple attribute decision making with hesitant fuzzy information. Experiments show very positive results.

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